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Effects of Radiation on Free Convection from a Heated Horizontal Circular Cylinder in the Presence of Heat Generation

(Kesan Radiasi terhadap Perolakan Bebas pada Silinder Membulat Melintang yang Dipanaskan dengan Penjana Haba)

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ABSTRACT

Effects of radiation on free convection about a heated horizontal circular cylinder in the presence of heat generation are investigated numerically. The cylinder is fixed and immersed in a stationary fluid, in which the temperature is uniformly heated about the temperature of the surrounding fluid. The governing equations are transformed into dimensionless non-linear partial differential equations and solved by employing a finite difference method. An implicit finite difference scheme of Crank Nicolson method is used to analyze the results. This study determined the effects of radiation parameter, R_d *heat generation parameter,* γ *and the Prandtl number, Pr on the temperature and velocity profiles. The results of the local heat transfer and skin-friction coefficient in the presence of radiation for some selected values of Pr and* γ *are shown graphically.*

Keywords: Free convection; heat generation; heated circular cylinder; radiation

ABSTRAK

Kesan radiasi pada perolakan bebas terhadap silinder bulat melintang dengan kehadiran haba yang dijana telah dikaji secara berangka. Silinder yang pegun direndam di dalam bendalir yang pegun dengan suhu silinder tersebut dipanaskan secara sekata berbanding suhu bendalir sekitar. Persamaan menakluk dijelmakan kepada persamaan terbitan separa tak linear dan diselesaikan menggunakan kaedah pembeza terhingga. Kaedah pembezaan terhingga tersirat Crank Nicolson telah digunakan. Kajian ini menentukan kesan parameter radiasi, R_d*, parameter penjana haba,* γ *dan nombor Prandtl, Pr terhadap profail suhu dan halaju. Hasil pemindahan haba setempat dan pekali geseran kulit dengan kehadiran radiasi untuk Pr dan* γ *tertentu ditunjukkan secara graf.*

Kata kunci: Penjana haba; perolakan bebas; radiasi; silinder bulat

INTRODUCTION

The effects of radiation on free convection boundary layer over a various shapes such as plate, sphere and cylinder have been studied among researchers because it is important in engineering applications, such as in advanced types of power plants for nuclear rockets, high-speed flights, re-entry vehicles and processes involving high temperature. The effect of heat generation in convection is also significant where there exist difference temperature between the surface and the ambient fluid. In certain applications on dealing with heat generation may alters the temperature and velocity distribution. Vajravelu (1979) is one of the earliest studies on heat generation effect to natural convection at heated semi-infinite vertical plate. Then, Vajravelu and Hadjinicolaou (1993) extended their previous studies in heat transfer characteristics in a laminar boundary layer flow of viscous fluid over a linearly stretching continuous surface with viscous dissipation or frictional heating and internal heat generation. In the above studies, they consider the volumetric of heat generation, $q'''[W/\mathrm{m}^3]$ as:

$$q''' = \begin{cases} Q_0(T^* - T^*_\infty, & \text{for } T^* \geq T^*_\infty \\ 0, & \text{for } T^* < T^*_\infty \end{cases},$$

where Q_0 is the heat generation and constant, T^* is the fluid temperature and T^*_∞ is the ambient temperature. The above relation is valid for the state of some exothermic processes having T^*_∞ as the onset temperature. Their work have been continued by other researchers such as Chamkha and Camile (2000) who studied the effects of heat generation or absorption on hydromagnetic flow over a flat surface. In addition, the study on the effect of heat generation over a plate has been done by Ferdousi and Alim (2010). Besides that, Molla et al. (2004) who studied on vertical wavy surface with uniform surface temperature has been extended by Molla et al. (2009a, 2006), both on a circular cylinder. Later, Cheng (2009) continued the study by Molla et al. (2009a, 2006) on a horizontal elliptical cylinder with constant heat flux. Recently, the study of the conjugate free convection from a horizontal circular cylinder in the presence of heat generation was investigated by Azim et al. (2012) and was extended to Azim and Chowdhury (2013) on the effect of heat generation and joule heating on laminar magnetohydrodynamic (MHD) conjugate natural convection flow from a horizontal circular cylinder.

Besides the presence of heat generation, the effects of radiation are also important. In the study on the effects of radiation on free convection was studied by Soundalgekar et al. (1960) which considered on free convection flow of a gas past a semi-infinite flat plate using the Cogley et al. (1968) equilibrium model. Hossain and Takhar (1996) analyzed the effects of radiation on free convection flow past a heated vertical plate. Later, Tahmina and Alim (2008) studied on a sphere with uniform surface heat. Besides that, Yih (1999) studied on a truncated cone by following the Rosseland diffusion approximation which also used by Molla et al. (2009b) to investigate the radiation effects on natural convection over vertical wavy frustum of cone. Then, Molla et al. (2011) extended their previous work to a horizontal circular cylinder. Furthermore, the study on free convection with the presence both of heat generation and radiation on natural convection flow over a various type of shape was investigated. The study on a sphere was investigated by Miraj and Alim (2010). Later, Miraj et al. (2011) investigate the effects of pressure work and radiation on their problem. Meanwhile Makinde (2012) studied on the hydromagnetic mixed convection stagnation point flow towards a vertical plate embedded in a highly porous medium. Ali et al. (2013) investigated MHD free convection flow along vertical flat plate. Ferdousi et al. (2013) studied on a porous vertical plate in the presence of heat generation. Recently, Elbashbeshy et al. (2014) was considered on the effect of heat generation or absorption and thermal radiation on free convection flow and heat transfer over a truncated cone in the presence of pressure work.

Ganesan and Rani (1998) presented numerical solution for the transient natural convection flow over a vertical cylinder under the combined buoyancy effects of heat and mass transfer by employing an implicit finite-difference method. Similar method has been applied by Rani and Devaraj (2003) in their study about unsteady flow past a vertical cylinder with temperature oscillations. A finite difference method is also presented in Abd El-Naby et al. (2004) work who studied radiation effects on unsteady MHD free convection flow over vertical porous plate. Rani and Kim (2008) demonstrated the Crank-Nicolson type of implicit finite- difference method in their study on the transient free convection flow over an isothermal vertical cylinder with temperature dependent viscosity. The above method was also used by Reddy and Reddy (2009a, 2009b) on the interaction of radiation and mass transfer effects on unsteady MHD free convection flow past a semi-infinite moving vertical cylinder. Recently, Udin and Harmand (2013) employed an unconditionally stable implicit finite difference scheme to solve their problem on unsteady natural convection heat transfer of nanofluid along a vertical plate embedded in porous medium and solved numerically by using explicit finite difference method that demonstrated in Carnahan et al. (1969) in study on free convection on a heated plate. The method has been used by Udin and Kumar (2010) in the problem of unsteady free convection in a fluid past an inclined plate immersed in a porous medium. Besides that, the method has also been employed by Khan et al. (2012) in his study on unsteady MHD free convection boundary- layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects.

The present study aimed to investigate the natural convection flow on a heated horizontal circular cylinder in the presence of heat generation and radiation by employing the Rosseland diffusion approximation. The governing equations of this study were transformed into dimensionless forms, which were solved numerically using a Crank-Nicolson type implicit finite-difference. The numerical results have been obtained in terms of local skin friction, rate of heat transfer, velocity profiles as well as temperature profiles for set of different radiation parameter, heat generation parameter and Prandtl number.

FORMULATION OF PROBLEM

Free convection flow from a heated horizontal circular cylinder in the presence of heat generation and radiation is considered. The flow over the horizontal circular cylinder is assumed to be two-dimensional, laminar, unsteady and incompressible. The physical configuration is shown in the Figure 1. T^*_s is assumed as the surface temperature of the cylinder, T^*_∞ is assumed as the ambient temperature of the fluid, where $T^*_s > T^*_\infty$ and T^* is the temperature of the fluid. By extending the problem of free convection on heated plate by Carnahan et al. (1969) and the problem of the effect of heat generation and radiation on free convection over the circular cylinder by Molla et al. (2006) and Miraj and Alim (2010), respectively, are considered. By considering the present physical conditions, usual Boussinesq approximation and boundary layer approximations, the full of continuity, momentum and energy equations are formulated in the following forms:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t^*}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta\left(T^*-T^*_\infty\right)\sin\left(\frac{x}{a}\right), \tag{2}$$

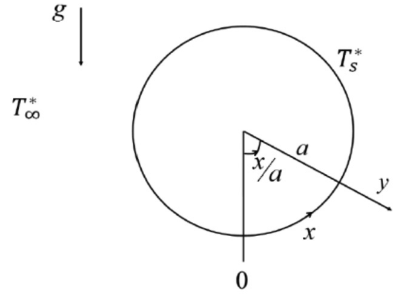

FIGURE 1. Physical model and coordinate system

$$\frac{\partial T^*}{\partial t^*}+u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial y}=\frac{\kappa}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^2}\right)-\frac{1}{\rho C_p}\left(\frac{\partial q_r''}{\partial y}\right)$$
$$+\frac{Q_0}{\rho C_p}\left(T^*-T^*_\infty\right), \quad (3)$$

where the x-coordinate is defined as a distance measured along the surface of a circular cylinder in vertically upward direction; y-coordinate is the outwards a distance measured normally from the surface of the cylinder; u and v denote the velocity components in the x and y directions, respectively; g is the gravitational acceleration is acting downward; and β is the coefficient of thermal expansion; (x/a) is the angle of the y-axis; κ is the thermal conductivity; ρ is the fluid density; C_p is the specific heat of constant pressure and Q_0 is a heat generation constant. While, q_r^* is the radiative heat flux in the y direction. The boundary conditions are taken as follows:

$$u=v=0,\ T^*=T^*_s \text{ at } y=0,$$
$$u\to 0,\ v\to 0,\ T^*\to T^*_\infty \text{ as } y\to\infty. \quad (4)$$

In (3), the term q_r'' is called as Rosseland diffusion approximation was obtained by Rosseland (1936) and is given by:

$$q_r''=\frac{4\sigma}{3\kappa\left(\alpha_r+\sigma_s\right)}\frac{\partial T^{*4}}{\partial y}, \quad (5)$$

where σ the Stefan-Boltzmann constant; α_r is Rosseland mean absorption coefficient; σ_s and is the scattering coefficient. The above expression that measures the radiative heat transfer, q_r'' is appropriate for two-dimensional boundary later flow. In order to non-dimensionalize (1)-(4) the following dimensionless variables were introduced:

$$X=\frac{x}{a},\ Y=\frac{y}{a}Gr^{1/4},\ t=\frac{a^2}{\upsilon}Gr^{-1/2}t^*,\ U=\frac{a}{\upsilon}Gr^{1/2}u,\ V=\frac{a}{\upsilon}Gr^{-1/4}v,$$
$$T=\frac{T^*-T^*_\infty}{T^*_s-T^*_\infty},\ Gr=g\beta\frac{\left(T^*_s-T^*_\infty\right)}{\upsilon^2}a^3,\ \gamma=\frac{Q_0a^2}{\mu C_\rho Gr^{1/2}},$$
$$T_s=\frac{T^*_s}{T^*_\infty},\ \Delta=T_s-1,\ R_d=\frac{4\sigma T^{*3}_\infty}{\kappa(\alpha_r+\sigma_s)},,\ \upsilon=\frac{\mu}{\rho},\ Pr=\frac{\mu C_\rho}{k}, \quad (6)$$

where υ is the reference kinematic viscosity; Gr is the Grashof number; γ is the heat generation parameter; T is non-dimensional temperature function; T_s is the surface temperature parameter; R_d is the radiation parameter; and Pr is the Prandtl number. Convert the governing (1)-(4) and the boundary conditions (4) into dimensional forms by substituting the non-dimensional variables above into (1)-(4) and becomes the following form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \quad (7)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+T\sin X, \quad (8)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial}{\partial Y^2}\left[\left\{1+\frac{4}{3}R_d\left(1+\Delta T\right)^3\right\}\frac{\partial T}{\partial Y}\right]+\gamma T. \quad (9)$$

The corresponding boundary conditions in dimensionless form are as follows:

$$U=V=0,\ T=1 \text{ at } Y=0,$$
$$U\to 0,\ V\to 0,\ T\to 0 \text{ as } Y\to\infty. \quad (10)$$

The physical quantities of principle interest are the rate of heat transfer and the shearing stress, τ_s in terms of the Nusselt number, Nu and skin-friction coefficient, C_f respectively, which can be written as

$$Nu=\frac{a\left(q_s''+q_r''\right)_{y=0}}{\kappa\left(T^*_s-T^*_\infty\right)},\ C_f=\frac{\left(\tau_s\right)_{y=0}}{\rho U_\infty^2}, \quad (11)$$

where $\tau_s=\mu(\partial u/\partial y)$ and $q_s''=-k(\partial T^*/\partial y)$. By using the non-dimensional variables (6) into (11) made the equation becomes in dimensionless forms:

$$NuGr^{-\frac{1}{4}}=-\left(1+\frac{4}{3}R_dT_s^3\right)\frac{\partial T}{\partial Y}\bigg|_{Y=0}, \quad (12)$$

$$C_fGr^{\frac{1}{4}}=X\frac{\partial^2 T}{\partial Y^2}\bigg|_{Y=0}. \quad (13)$$

An implicit finite difference scheme of Crank Nicolson type is employed to solve the non-linear partial differential equations (7)-(9) under the boundary conditions (10). The region of solution is considered from the stagnation point $X=0$ to $X_{max}=\pi$ and an axis normal to the surface from $Y=0$ to $Y_{max}=30.0$ as corresponding to Y_∞. The subscripts i and j designate the grid points along the X and Y coordinates, respectively, where $X_i=i\Delta X$ and $Y_j=j\Delta Y$ and the subscript k designates the value of time $t=k\Delta t$ with the mesh sizes $\Delta X=\pi/150$, $\Delta Y=0.12$ with the time step size dependency of $\Delta t=0.01$. The iterative procedure is stopped when the time step reached to $t_{max}=10$. (7)-(9) subject to the boundary conditions (10) are discretized using Crank-Nicolson type of implicit finite difference method and finally the system of tri-diagonal system of equations are obtained. Such a system of equations was solved by Thomas algorithm as described by Carnahan et al. (1969).

NUMERICAL RESULTS

The numerical results of study on free convection on a heated horizontal circular cylinder in the presence of heat generation are presented. This work was considered the effects of radiation and heat generation parameter on the rate of heat transfer, the skin-friction coefficient, the temperature and velocity profiles. The effect of different Prandtl number will also be presented. Here, the Prandtl numbers used are Pr = 0.7, 4.0, 7.0 and the effects of radiation parameter R_d = 0.0, 0.2, 0.5, 1.0 while, the heat generation parameter γ = 0.0, 0.5, 1.0. The numerical results of the local heat transfer or Nusselt number and the local skin-friction coefficient shown in Table 1 represented the comparisons between of the present study with the previous work by Molla et al. (2011). Here, Prandtl number, Pr = 0.73, heat generation parameter, γ = 0.0 have been chosen. It is observed from the table that the present result has a good agreement with the results of Molla et al. (2011).

The effects of radiation parameter R_d on the velocity and temperature profiles with Pr = 0.7, γ = 0.0 and surface temperature parameter T_s = 1.0 are shown in Figures 2 and 3. Here, as R_d increases, both the velocity and the temperature increase because the thermal boundary layer thickness increases and the thickness of the velocity boundary layer also increase. Which means the velocity and temperature gradients at the surface increase enhances the fluid velocity and temperature. The local maximum of the velocity exist within the boundary layer, but velocity increases near the surface of the cylinder and then decreases slowly approach to zero while the temperature from the surface decreases slowly and finally tends to zero. The change of velocity and temperature profiles in the Y direction satisfied the natural convection boundary layer flow.

Figures 4 and 5 show the local rate of heat transfer Nu and the local skin-friction coefficient C_f respectively, increase for increasing values of R_d, when Pr = 0.7 and γ = 0.0 when t = 4.0. It is found that from the Figure 5, the skin friction increase gradually from zero at beginning of stagnation point X = 0 along the X direction and slowly decrease to downstream point $X_{max} = \pi$. From Figure 4, it shows that the rate of heat transfer decrease slightly along the X direction from beginning stagnation point to the downstream.

The variation of Pr on the velocity and temperature profiles when γ = 0.0 and R_d = 0.2 at t = 4.0 are shown in the Figure 6. It was found that as the Pr increase, then both of the temperature and velocity decrease as well as the thermal boundary layer thickness decrease significantly. Variations of the local skin friction coefficient C_f and local

TABLE 1. Comparison between the present results and Molla et al. (2011) for the rate of heat transfer and the local skin friction coefficients when T_s = 1.1, R_d = 0.5, γ = 0.0, and Pr = 0.73

X	$NuGr^{-1/4}$		$G_f Gr^{1/4}$	
	Present study	Molla et al. (2011)	Present study	Molla et al. (2011)
0	0.54492	0.54424	0.00000	0.00000
$\pi/6$	0.53773	0.53776	0.46937	0.46890
$\pi/3$	0.51852	0.51845	0.86103	0.85457
$\pi/2$	0.48592	0.48592	1.07755	1.08646
$2\pi/3$	0.43871	0.43887	1.11688	1.11449
$5\pi/6$	0.37512	0.37391	0.92132	0.91093
π	0.27337	0.27466	0.44234	0.44387

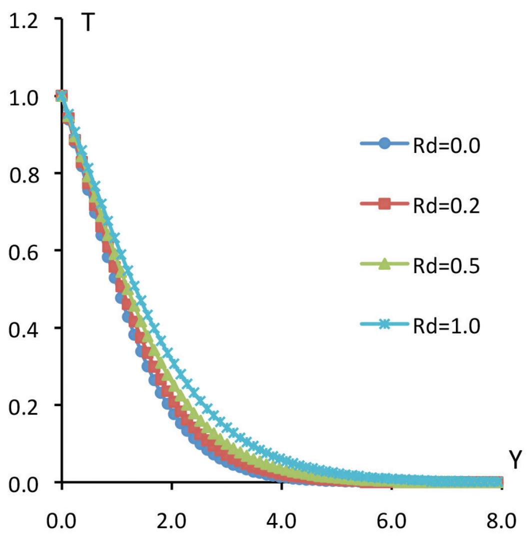

FIGURE 2. Temperature profile against Y with γ = 0.0 and Pr = 0.7 at t = 4.0 and $X = \pi/6$

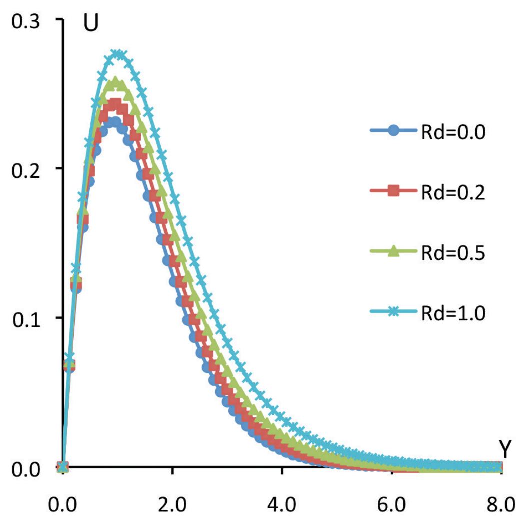

FIGURE 3. Velocity profile against Y with γ = 0.0 and Pr = 0.7 at t = 4.0 and $X = \pi/6$

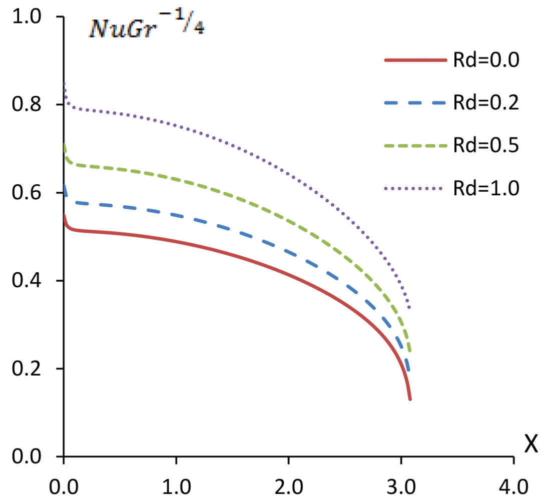

FIGURE 4. Local Nusselt number with γ = 0.0 and Pr = 0.7 at t = 4.0

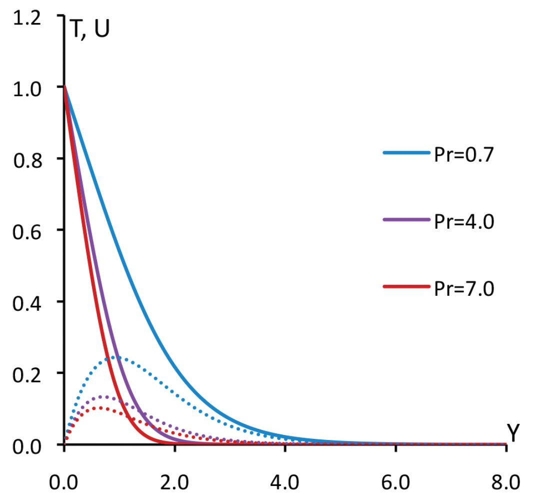

FIGURE 6. Temperature and velocity profiles against Y with R_d = 0.2 and Pr = 0.7 at t = 4.0 and $X = \pi/6$

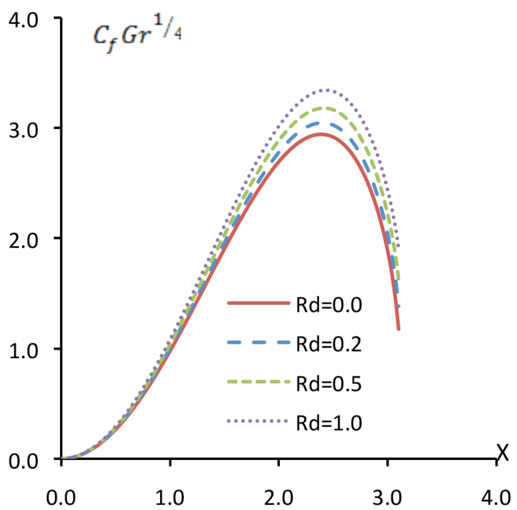

FIGURE 5. Local skin friction with γ = 0.0 and Pr = 0.7 at t = 4.0

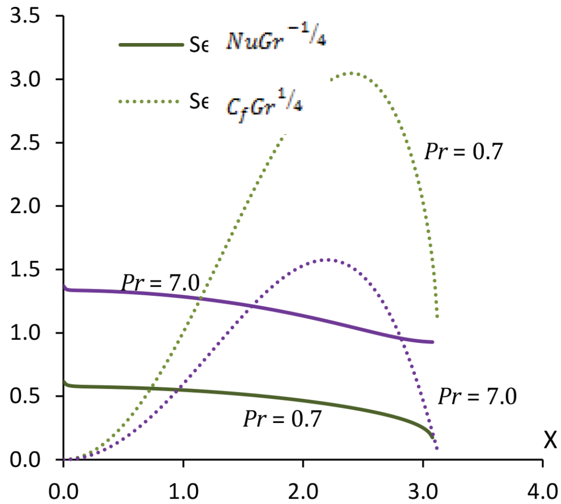

FIGURE 7. Local skin friction and Nusselt number with γ = 0.0 and R_d = 0.2 at t = 4.0 and $X = \pi/6$

Nusselt number Nu for different values of Pr when $\gamma = 0.0$ and $R_d = 0.2$ at $t = 4.0$ are shown in Figure 7. It was found that as Pr increase, the skin friction coefficient decrease and heat transfer coefficient increase.

The comparison between the study in the presence of heat generation, the presence of radiation and the presence of both radiation and heat generation on the temperature profiles against time, t at various positions of Y *is* plotted in Figure 8 with $Pr = 0.7$, $\gamma = 0.2$ for the presence of heat generation and $R_d = 0.2$ for the presence of radiation. It was observed that the temperature profile increase from the beginning of time then becomes steady. Means, after time t = 3.0, the temperature of fluids does not change or remain constant after reaching the maximum temperature level. Besides that, the value of the temperature in the presence of radiation is lower than the presence of heat generation while, by presenting both of radiation and heat generation made the temperature becomes higher. It is because the heat generated internally within the surface of cylinder more affected rather than the presence of radiation from the outside of the surface cylinder.

The effects of radiation parameter R_d and the heat generation parameter on the temperature and velocity profiles with Pr = 0.7 are illustrated in Figures 9 and 10. Figure 10 shows that the velocity profiles increase as increasing the values of γ and R_d. The temperature profiles in Figure 8 increase as γ increase. While, as R_d increases with the increasing of γ, the temperature profiles near the surface decrease and then as away from the surface the temperature profiles becomes higher.

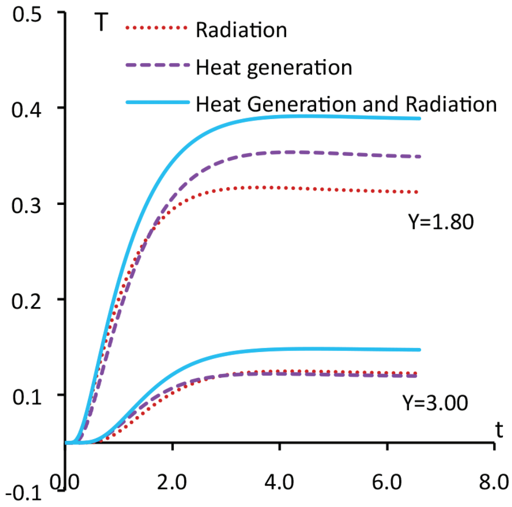

FIGURE 8. Temperature profiles against time, t with $Pr = 0.7$, $\gamma = 0.2$ for the presence of heat generation and $R_d = 0.2$ for the presence of radiation at $X = \pi/6$

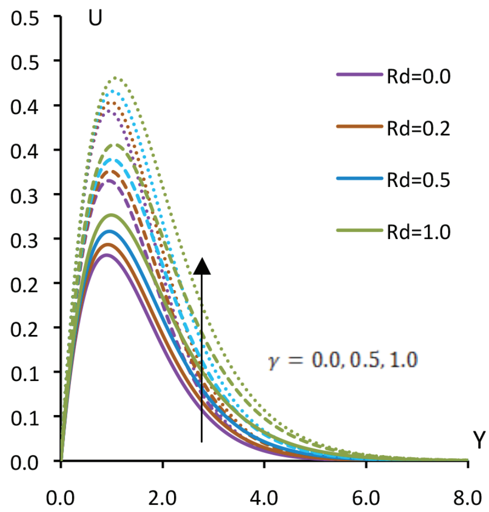

FIGURE 10. Velocity profile against Y with $\gamma = 0.0, 0.5, 1.0$ and $Pr = 0.7$ at $t = 4.0$ and $X = \pi/6$

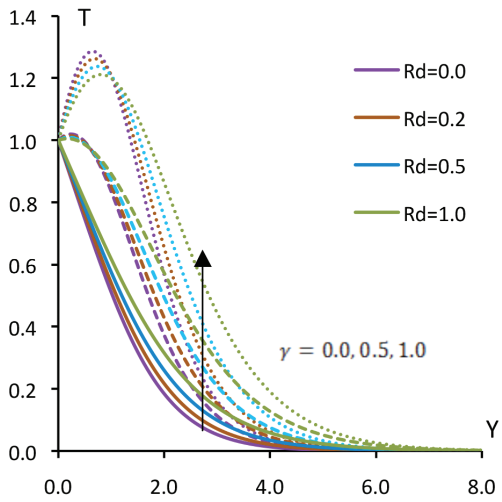

FIGURE 9. Temperature profile against Y with $\gamma = 0.0, 0.5, 1.0$ and $Pr = 0.7$ at $t = 4.0$ and $X = \pi/6$

CONCLUSION

The effects of radiation on natural convection flow on a heated horizontal cylinder in the presence of heat generation have been investigated for different values of relevant physical parameters. From the results presented, the comparison between the present results and the previous study was found to be in good agreement. The radiation parameter increased with the increase of velocity, temperature, local skin friction coefficient and local rate of heat transfer. The increase in Prandtl number leads to decrease of velocity, temperature, local skin friction coefficient but with the increase of the local rate of heat transfer. This study also found the existence of steady flow around the circular cylinder. The temperature profiles with the presence of both heat generation and radiation are higher than the presence either a heat generation or radiation. Moreover, the effect of heat generation has significantly influence the fluid temperature compared to the presence of radiation only.

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REFERENCES

Abd El-Naby, M.A., Elsayed, M.E.E & Nader, Y.A. 2003. Finite difference solution of radiation effects on MHD unsteady free convection flow over vertical porous plate. *Applied Mathematics and Computation* 151: 327-346.

Ali, M.M., Mamun, A.A., Maleque, M.A. & Azim, NHM. A. 2013. Radiation effects on MHD free convection flow along vertical flat plate in presence of Joule heating and heat generation. *Proceeding of the Fifth BSME International Conference on Thermal Engineering* 56: 503-509.

Azim, NHM. & Chowdhury, M.K. 2013. MHD-conjugate free convection from an isothermal horizontal circular cylinder with joule heating and heat generation. *Journal of Computational Methods in Physics* 2013: Article ID 180516.

Azim, NHM., Morshed, M. & Chowdhury, M.K. 2012. Conjugate free convection from a horizontal isothermal circular cylinder with heat generation. *ASA University Review* 6(2): 145-154.

Carnahan, B., Luther, H.A. & Wilkes, J.O. 1969. *Applied Numerical Methods*. New York: Wiley.

Chamkha, A.J. & Camile, I. 2000. Effects of heat generation or absorption and the thermophoresis on hydromagnetic flow with heat and mass transfer over a flat plate. *International Journal of Numerical Methods for Heat & Fluid Flow* 10(4): 432-438.

Cheng, C.Y. 2009. Natural convection heat transfer from a horizontal isothermal elliptical cylinder with internal heat generation. *International Communications in Heat and Mass Transfer* 36(4): 346-350.

Cogley, A.C., Vincenti, W.G. & Giles, S.E. 1968. *Differential Approximation for Radiation Transfer in a Non Gray Near Equilibrium*. New York: Springer.

Elbashbeshy, E.M.A., Emam, T.G. & Sayed, E.A.A. 2014. Effect of thermal radiation on free convection flow and heat transfer over a truncated cone in the presence of pressure work and heat generation/absorption. *Thermal Science*. DOI: 10.2298/TSCI130409006E.

Ferdousi, A., Rahman, M.M., Parvez, M.S. & Alim, M.A. 2013. Effect of radiation on natural convection flow from a porous vertical plate in presence of heat generation. *International Journal of Engineering and Applied Sciences* 3(4): 20-29.

Ferdousi, A. & Alim, M.A. 2010. Natural convection flow from a porous vertical plate in presence of heat generation. *International University Journal of Science and Technology* 5(1): 73-80.

Ganesan, P. & Rani, H.P. 1998. Transient natural convection flow over vertical cylinder with heat and mass transfer. *Heat and Mass Transfer* 33: 449-455.

Hossain, M.A. & Takhar, H.S. 1996. Radiation effect on mixed convection along a vertical plate with uniform surface temperature. *Heat and Mass Transfer* 31(4): 243-248.

Khan, M.S., Karim, I., Ali, L.E. & Islam, A. 2012. Unsteady MHD free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects. *International Nano Letters* 2(24): 1-9.

Miraj, M., Alim, M.A. & Andallah, L.S. 2011. Effects of pressure work and radiation on natural convection flow around a sphere with heat generation. *International Communications in Heat and Mass Transfer* 38(7): 911-916.

Miraj, M. & Alim, M.A. 2010. Effect of radiation on natural convection flow on a sphere in presence of heat generation. *International Communications in Heat and Mass Transfer* 37(6): 660-665.

Makinde, O.D. 2012. Heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation. *Meccanica* 47(5): 1173-1184.

Molla, M.M., Saha, S.C., Khan, M.A.I. & Hossain, M.A. 2011. Radiation effects on natural convection laminar flow from a horizontal circular cylinder. *Desalination and Water Treatment* 30(1-3): 89-97.

Molla, M.M., Hossain, M.A. & Paul, M.C. 2009a. Natural convection flow from a horizontal circular cylinder with uniform heat flux in presence of heat generation. *Applied Mathematic Modelling* 33(7): 3226-3236.

Molla, M.M., Hossain, M.A. & Gorla, R.S.R. 2009b. Radiation effect on natural convection boundary layer flow over a vertical wavy frustum of cone. *Journal of Mechanical Engineering Science* 223(7): 1605-1614.

Molla, M.M., Hossain, M.A. & Paul, M.C. 2006. Natural convection flow from an isothermal horizontal circular cylinder in presence of heat generation. *International Journal of Engineering Sciences* 44(13-14): 949-958.

Molla, M.M., Hossain, M.A. & Lun Shin Yao. 2004. Natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation or absorption. *International Journal of Thermal Sciences* 43(2): 157-163.

Rani, H.R. & Kim, C.N. 2008. Transient free convection flow over an isothermal vertical cylinder with temperature dependent viscousity. *Korean Journal Chemical Engineering* 25(1): 34-40.

Rani, H.R. & Devaraj, R. 2003. Numerical solution of unsteady flow past a vertical cylinder with temperature oscillations. *Foshung in Ingenieurwesen* 68(2): 75-78.

Reddy, M.M.G. & Reddy, N.B. 2009a. Radiation and mass transfer effects on unsteady MHD free convection flow of an incompressible viscous fluid past a moving vertical cylinder. *Theoretical Applied Mechanics* 36(3): 239-260.

Reddy, M.M.G. & Reddy, N.B. 2009b. Thermal radiation and mass transfer effects on MHD free convection flow of an vertical cylinder with variable surface temperature and concentration. *Journal of Naval Architecture and Marine Engineering* 6(1): 1-24.

Soundalgekar, V.M., Takhar, H.S. & Vighnesam, N.V. 1960. The combined free and forced convection flow past a semi-infinite vertical plate with variable surface temperature. *Nuclear Engineering and Design* 110(1): 95-98.

Tahmina, A. & Alim, M.A. 2008. Effects of radiation on natural convection flow around a sphere with uniform surface heat flux. *Journal of Mechanical Engineering* 39(1): 50-56.

Udin, Z. & Harmand, S. 2013. Natural convection heat transfer of nanofluids along a vertical plate embedded in porous medium. *Nanoscale Research Letters* 8: 64.

Udin, Z. & Kumar, M. 2010. Unsteady free convection in a fluid past an inclined plate immersed in a porous medium. *Computer Modelling and New Technologies* 14(3): 41-47.

Vajravelu, K. & Hadjinicolaou, A. 1993. Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. *International Communication in Heat and Mass Transfer* 20(3): 417-430.

Vajravelu, K. 1979. Natural convection at a heated semi infinite vertical plate with internal heat generation. *Acta Mechanica* 34(1-2): 153-159.

Yih, K.A. 1999. Effect of radiation on natural convection about a truncated cone. *International Journal of Heat and Mass Transfer* 42(23): 4299-4305.

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